Turkish Journal of Engineering

BUCKLING ANALYSIS OF LAMINATED COMPOSITE PLATES UNDER THE EFFECT OF UNIAXIAL AND BIAXIAL LOADS

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ABSTRACT

This paper investigates the buckling analysis of simply supported symmetrically thin and thick composite plates. Using the Hamilton's principle, the governing equation for thin and thick composite plates is derived. The equation of motion for thin and thick laminated rectangular plates subjected to in-plane loads is obtained with the help of Hamilton's principle. The loading conditions of rectangular plate are uniaxial and biaxial compression. Considering the Navier solution technique, closed form solutions are attained and buckling loads are found by solving the eigenvalue problems. In this study, the effect of edge ratios and anisotropy on the buckling analysis of rectangular plate was investigated. The computer programs have been written separately with the help of Mathematica (MATHEMATICA 2017) program for the solution of the buckling analysis of laminated composite plates. Results of the numerical studies for the buckling of laminated composite plates (LCP) are demonstrated and benchmarked with former studies in the literature and ANSYS finite element methods.

Keywords: Buckling Analysis, Laminated Composite Plate, Shear Deformation Theory, Finite Element Method (ANSYS).

1. INTRODUCTION

Recently, due to the many paramount properties advanced composite materials such as laminated plates are found an application area in the engineering projects. Tremendous researches have been performed on the LCP to clarify the advantages of using these types of materials. One of the focused topics in research subject is the buckling analysis of the composite plates.

Reissner theory (1945) is one of the theories which include the shear deformation effect and many researchers have studied on the buckling analysis of LCP by using Reissner theory. Noor (1975) examined the stability and vibration analysis of the composite plates. Qatu used energy function to develop governing equations of LCP. Phan and Reddy (1985) are analyzed of laminated composite plates using a higher-order shear deformation theory. Reddy and Khdeir (1989) investigated buckling and vibration analysis of LCP. Some studies have been performed on characteristics of plates by Qatu (1991-2004) using different plate theories. Dogan et al. (2010) have analyzed the effects of anisotropy and curvature on vibration characteristics of laminated shallow shells using shear deformation theory. Dogan (2012) investigated the effect of dimension on mode-shapes of composite shells. Akavci (2007) presented buckling and free vibration analysis of symmetric and antisymmetric laminated composite plates on an elastic foundation. Akavci et al. (2007) examined buckling and free vibration behavior of LCP on elastic foundation by using first order shear deformation theory (FSDT). Functionally graded plates thermal buckling analysis have been investigated by Akavci (2014) using the theory of hyperbolic shear deformation. Phan and Reddy (1985) analyzed laminated composite plates by using a higher-order shear deformation theory. Setodehand Karami (2004) studied on buckling analysis of laminated composite plates of elastic foundation. Sophy (2013) studied buckling and free vibration of exponentially graded sandwich plates resting on elastic foundations under various boundary conditions. Dogan (2019) investigated buckling analysis of symmetric laminated composite thick plates. Sayyad and Ghuga (2014) presented a study about buckling and free vibration analysis of orthotropic plates by using exponential shear deformation theory.

In this research, buckling analysis of symmetric LCP are investigated using various of number of layers, plate edge ratio and anisotropy ratio. This study might be a pioneer work in terms of laminated composite plates and experimental studies.

2. EQUATIONS

A lamina is produced with the isotropic homogenous fibers and matrix materials (Fig. 1). Any point on a fiber, and/or on matrix and/or on matrix-fiber interface has crucial effect on the stiffness of the lamina. Due to the big variation on the properties of lamina from point to point, macro-mechanical properties of lamina are determined based on the statistical approach.
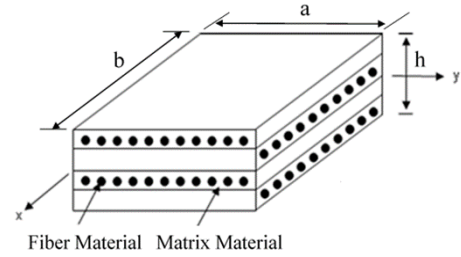

Fig. 1. Fiber and matrix materials in laminated composite plate

According to FSDT, the transverse normal does not cease perpendicular to the mid-surface after deformation. It will be assumed that the deformation of the plates is completely determined by the displacement of its middle surface. Using the given equation below (Eq.1) nth layer lamina plate stress-strain relationship can be defined in lamina coordinates (Qatu 2004).

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{13} & 0 & 0 & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{23} & 0 & 0 & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{33} & 0 & 0 & \bar{Q}_{36} \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} & 0 \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{36} & 0 & 0 & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix} \quad (1)$$

The displacement based on plate theory can be written as

$$u(x, y, z, t) = u_0(x, y, t) + z\,\varphi_x(x, y, t)$$
$$v(x, y, z, t) = v_0(x, y, t) + z\,\varphi_y(x, y, t) \quad (2)$$
$$w(x, y, z, t) = w_0(x, y, t)$$

where u, v, w, φ_x and φ_y are displacements and rotations in x, y, z direction, orderly. u_o, v_o and w_o are mid-plane displacements.

Equation of motion for plate structures can be derived by Hamilton's principle

$$\delta \int_{t_1}^{t_2} (T + W - U)\,dt = 0 \quad (3)$$

where T is the kinetic energy of the structure

$$T = \frac{\rho}{2} \int \left\{ \left(\frac{\partial u}{\partial t} \right)^2 + \left(\frac{\partial v}{\partial t} \right)^2 + \left(\frac{\partial w}{\partial t} \right)^2 \right\} dxdydz \quad (4)$$

W is the work of the external forces

$$W = \iint_{x\,y} (q_x u + q_y v + q_z w + m_x \psi_x + m_y \psi_y)\,dxdy \quad (5)$$

in which, q_x, q_y, q_z, m_x, m_y are the external forces and moments per unit length, respectively. U is the strain energy and defined as,

$$U = \frac{1}{2} \int \big(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z + \tau_{xy} \gamma_{xy} + \tau_{xz} \gamma_{xz} + \tau_{yz} \gamma_{yz} \big) dxdydz \quad (6)$$

Solving equation 3 gives set of equations called equations of motion for plate structures. This gives equation 7 in simplified form as,

$$\frac{\partial}{\partial x} N_x + \frac{\partial}{\partial y} N_{yx} + q_x = I_1 \ddot{u}^2 + I_2 \ddot{\psi}_x^2$$

$$\frac{\partial}{\partial y} N_y + \frac{\partial}{\partial x} N_{xy} + q_y = I_1 \ddot{v}^2 + I_2 \ddot{\psi}_y^2 \qquad (7)$$

$$\frac{\partial}{\partial x} Q_x + \frac{\partial}{\partial y} Q_y + q_z + k_0 w + k_1 \Delta^2 w = I_1 \ddot{w}^2$$

$$\frac{\partial}{\partial x} M_x + \frac{\partial}{\partial y} M_{yx} - Q_x + m_x = I_2 \ddot{u}^2 + I_3 \ddot{\psi}_x^2$$

$$\frac{\partial}{\partial y} M_y + \frac{\partial}{\partial x} M_{xy} - Q_y + m_y = I_2 \ddot{v}^2 + I_3 \ddot{\psi}_y^2$$

Equation 7 is defined as equation of motion for thick plates. Where I_1, I_2 and I_3 are mass moment inertia. Based on the FSDT plate, moment resultants are M_x, M_y and M_{xy}, in plane force resultants are N_x, N_y, N_{xy} and transverse shearing force resultants are Q_x and Q_y. The force and moment resultants are defined as

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{0x} \\ \varepsilon_{0y} \\ \gamma_{0xy} \\ \kappa_x \\ \kappa_y \\ \tau \end{bmatrix}$$

$$(8)$$

$$\begin{bmatrix} Q_x \\ Q_y \end{bmatrix} = \begin{bmatrix} A_{55} & A_{45} \\ A_{45} & A_{44} \end{bmatrix} \begin{bmatrix} \gamma_{0xz} \\ \gamma_{0yz} \end{bmatrix}$$

$$\{A_{ij}, B_{ij}, D_{ij}\} = \int_{-h/2}^{h/2} \{1, z, z^2\} \overline{Q}_{ij} dz \quad i, j = 1,2,6$$

$$\{A_{ij}\} = K_S \int_{-h/2}^{h/2} \overline{Q}_{ij} dz \qquad i, j = 4,5 \qquad (9)$$

$$[I_1, I_2, I_3] = \int_{-h/2}^{h/2} \rho(1, z, z^2) dz$$

Where the parameter K_s is the shear correction factor. Here, K_s is taken as 5/6.

The Navier type solution might be implemented to thick and thin plates. This type solution assumes that the displacement section of the plates can be denoted as sine and cosine trigonometric functions. A plate with shear diaphragm boundaries on all edges is assumed. For simply supported thick plates, boundary conditions can be arranged as follows:

$$N_x = w_0 = v_0 = M_x = \psi_y = 0 \qquad x = 0, a$$
$$N_y = w_0 = u_0 = M_y = \psi_x = 0 \qquad y = 0, b \qquad (10)$$

The displacement functions of the satisfied the boundary conditions are applied;

$$u_0(x,y,t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} U_{mn} \cos(x_m x) \sin(y_n y) \sin(\omega_{mn} t)$$

$$v_0(x,y,t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} V_{mn} \sin(x_m x) \cos(y_n y) \sin(\omega_{mn} t)$$

$$w_0(x,y,t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} W_{mn} \sin(x_m x) \sin(y_n y) \sin(\omega_{mn} t)$$
$$(11)$$

$$\psi_x(x,y,t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \psi_{xmn} \cos(x_m x) \sin(y_n y) \sin(\omega_{mn} t)$$

$$\psi_y(x,y,t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \psi_{ymn} \sin(x_m x) \cos(y_n y) \sin(\omega_{mn} t)$$

where $x_m = m\pi/a$, $y_n = n\pi/b$. and ω_{mn} is the naturel frequency. The loads are defined as,

$$q(x,y,t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Q_{mn}(t) \sin(x_m x) \cos(y_n y) \qquad (12)$$

where

$$Q_{mn}(t) = \frac{4}{ab} \int_0^a \int_0^b q(x, y, t) \sin(x_m x) \cos(y_n y) dx\, dy \quad (13)$$

Substituting the above equations into the equation of motion in matrix form,

$$[N] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha^2 \overline{N}_x + \beta^2 \overline{N}_y & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (14)$$

$$[K] = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} \end{bmatrix} \qquad (15)$$

$$[\Delta] = \begin{bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ \psi_{xmn} \\ \psi_{ymn} \end{bmatrix} \qquad (16)$$

Equation 12 can be arranged in a closed form as follows:

$$([K_{mn}] - \omega[N])\Delta = 0 \tag{17}$$

where $[K_{mn}]$, stiffness matrices, $[N]$ is buckling load.

3. NUMERICAL SOLUTIONS AND DISCUSSIONS

In current research, buckling analyses of symmetric LCP are investigated. Navier solution procedure for buckling analysis of LCP is obtained. The computer programs have been prepared using Mathematica program separately for the solution of the buckling analysis of LCP. The results were compared with the semi-analytical method and the ANSYS finite element software and previous studies in the literature. The effects of the E_1/E_2, and a/b ratio are also investigated.

In numerical calculations, the material and geometrical properties are defined as:

$a = 1m$ (a/b = 1, 2; a/h = 10, $\rho = 2000$ kg/m^3, $E_1 = 40 \times 10^3$ MPa ($E_1/E_2 = 3$, 10, 20, 30, 40), $G_{12}/E_2 = G13/E2 = 0.6$, $G_{23}/E_2 = 0.5$, $\upsilon = 0.25$. In the analysis, following parameter is studied for non-dimensional buckling load as;

$$\lambda = \overline{N_x} \frac{a^2}{E_2 h^3} \tag{18}$$

It can be seen from Tables 1 that the non-dimensional buckling load factors increase when the ratio of E_1/E_2 change from 3 to 40 (Fig. 3). The non-dimensional buckling load factors decrease when the ratio of a/b change from 1 to 2 for Table 2 and Fig 4-5. Also, the non-dimensional buckling load factors obtained for present study seem to be compatible with other study. Buckling analysis results showed that when the number of layer increases, the non-dimensional buckling load factors obtained by present study increase as well (Fig. 6-7).
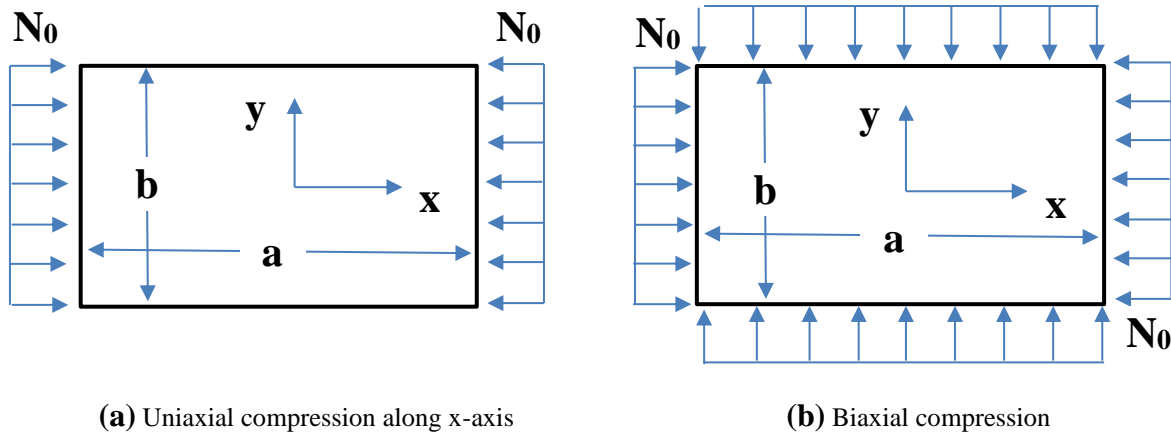
(a) Uniaxial compression along x-axis

(b) Biaxial compression

Fig. 2. The loading conditions of plate

Table 1. Unaxial non-dimensional buckling load factors of (0/90/90/0) laminate for different orthotropy ratios (a/b=1 and a/h=10)

Method	E_1/E_2				
	3	10	20	30	40
CLPT [Phan etc.]	5.7538	11.4920	19.7120	27.9360	36.1600
HSDT[Phan etc.]	5.1143	9.7740	15.2980	19.9570	23.3400
3DElasticity[Noor]	5.2944	9.7621	15.0191	19.3040	22.8807
ANSYS (FEM)	7.2290	11.2470	16.2720	19.4090	23.8800
Akavci	5.4192	10.0671	16.6358	19.3040	24.1601
Present Study	5.4550	10.0056	15.3828	19.7824	23.4746

Table 2. Unaxial and biaxial non-dimensional buckling load factors of (0/90/0) rectangular plate (b/h= 10 and E_1/E_2=40)

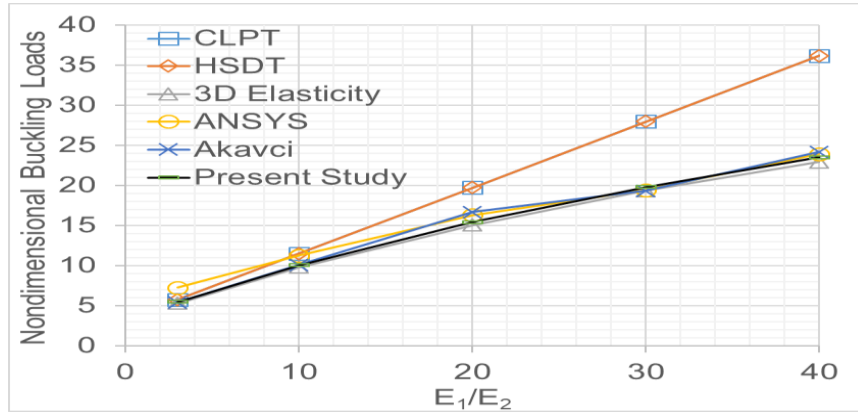| a/b	Unaxial				Biaxial			
	Setodeh	Akavci	Ansys [Fem]	Present Study	Setodeh	Akavci	Ansys [Fem]	Present Study
1	22.234	22.115	22.514	22.334	9.942	9.953	9.955	10.208
2	16.424	16.308	18.122	16.450	3.269	3.261	3.328	3.290

Fig. 3. Effect of orthotropy ratio on the uniaxal buckling load factor of a square symmetrically [0/90/90/0] laminated plate.
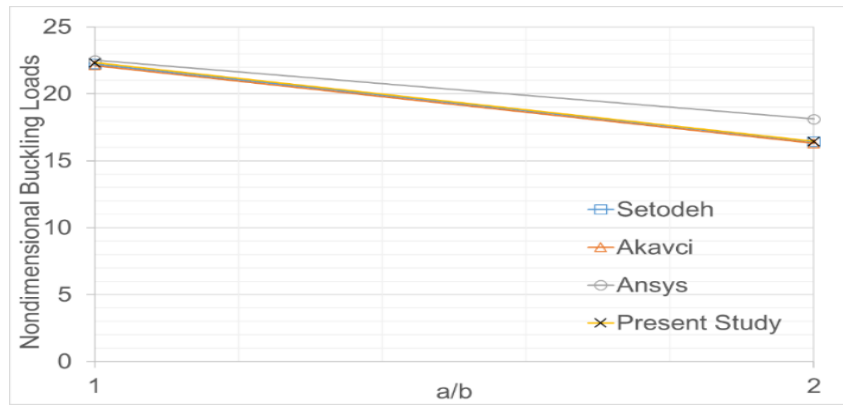

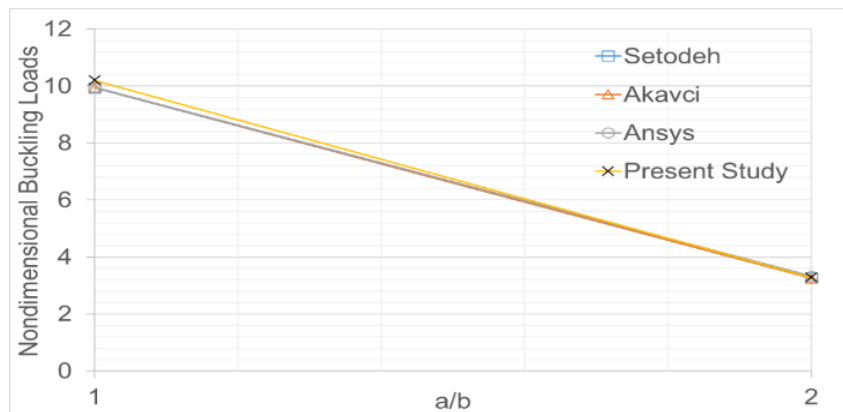Fig. 4. Effect of edge ratio on the uniaxal buckling load factor of [0/90/0] laminated plate.

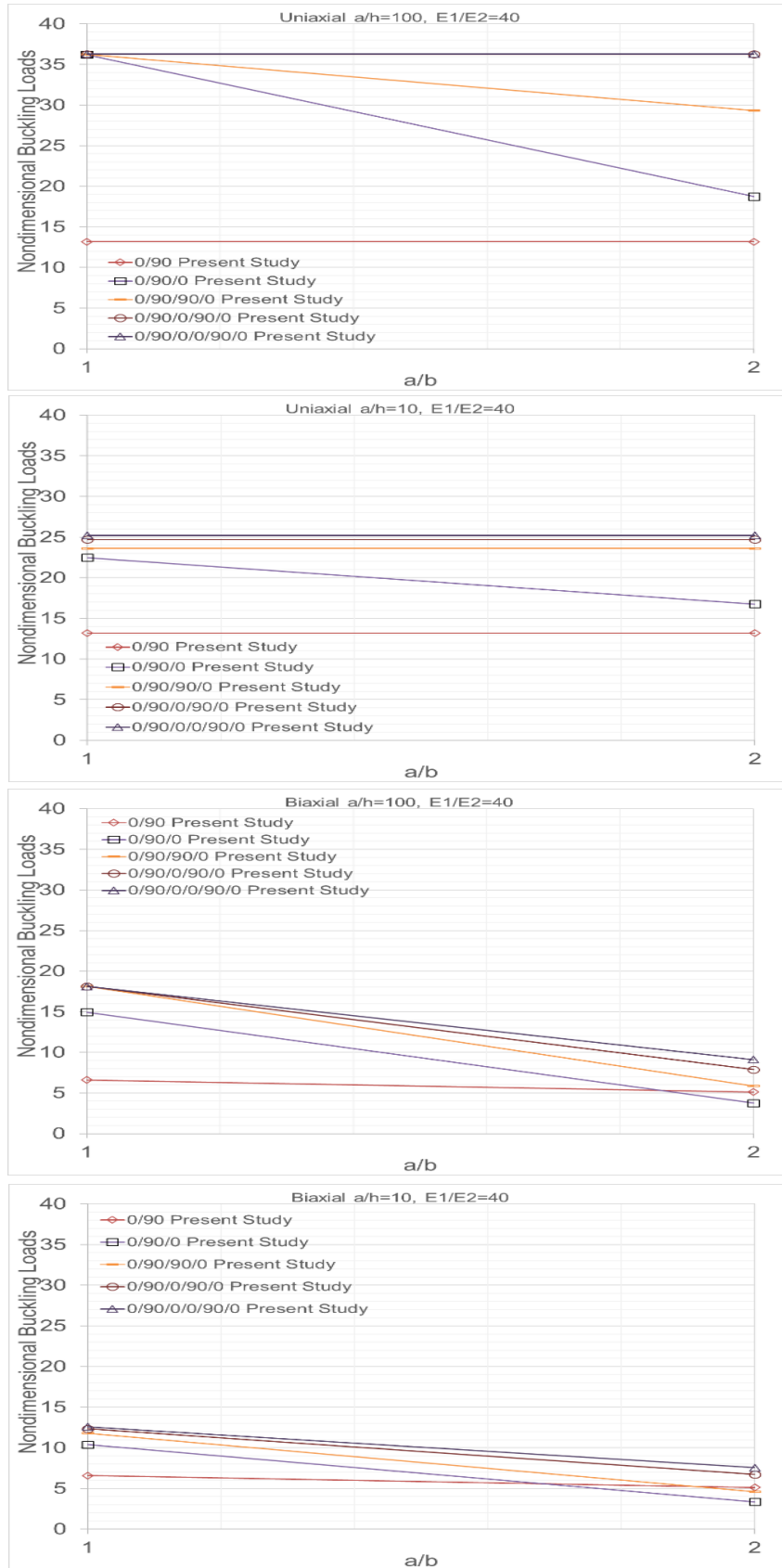Fig. 5. Effect of edge ratio on the biaxal buckling load factor of [0/90/0] laminated plate.

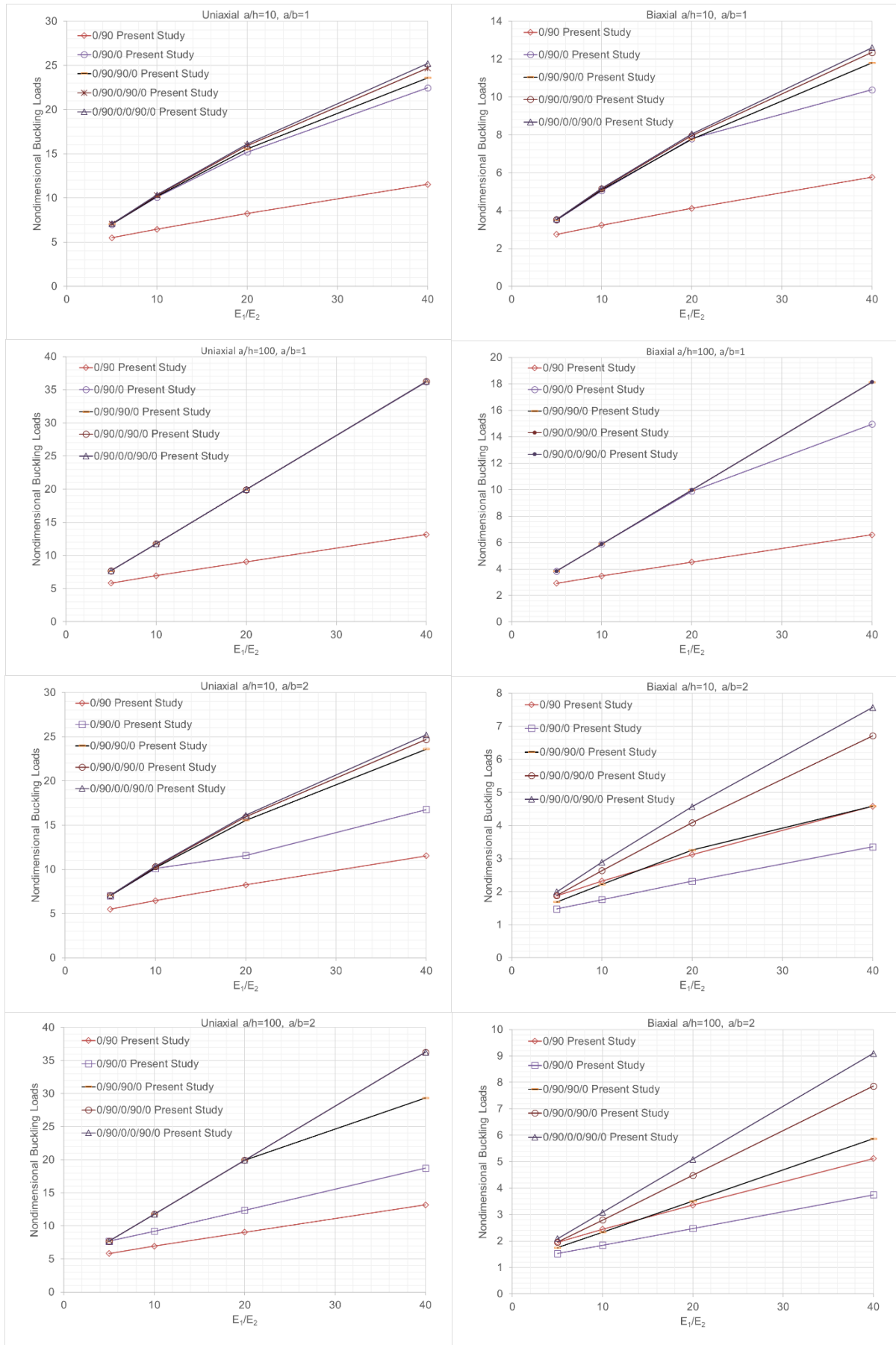Fig. 6. Effect of edge ratio on the uniaxial and biaxal buckling load factor of various laminated sequence.

Fig. 7. Effect of anisotropy ratio on the uniaxial and biaxal buckling load factor of various laminated sequence

4. CONCLUSION

In this study, buckling analysis of symmetrically (LCP) resting is theoretically investigated. By applying Hamilton's principle, the governing equation for thick LCP is obtained. The solutions are gathered by using Navier solution method. The effects of a/b, E_1/E_2 ratios on buckling loads are examined. The most important observations and results are summarized as follows:

- The non-dimensional buckling load factors obtained for present study seem to be compatible with other study.
- The a/b, E_1/E_2 ratios, are playing a crucial role on buckling loads.
- The non-dimensional buckling load factors generally decrease when the ratio of a/b change from 1 to 2.
- The non-dimensional buckling load factors increase when the ratio of E_1/E_2 change from 3 to 40.
- The number of layer increases, the non-dimensional buckling load factors obtained by present study increase as well.

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