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Nonlinear Chaotic Analysis of USD/TRY and EUR/TRY Exchange Rates

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DOLAR/TL ve EURO/TL Döviz Kurlarının Doğrusal Olmayan ve Kaotik Analizi	**Nonlinear Chaotic Analysis of USD/TRY and EUR/TRY Exchange Rates**
Öz	**Abstract**
Bu çalışmada DOLAR/TL ve EURO/TL döviz kurları doğrusal olmayan ve kaotik zaman serileri analizi yöntemleriyle analiz edilmiştir. Bu çalışmada kaosun tespiti için korelasyon boyutu, Lyapunov katsayısı, vekil veri testi yöntemleri kullanılmış ve DOLAR/TL ve EURO/TL döviz kurlarında kaosun bulunduğuna dair kanıtlar elde edilmiştir. Ayrıca yineleme niceleme analizi (YNA) ve çapraz yineleme niceleme analizi (ÇYNA) yöntemleri kullanılarak döviz kurlarının kaotik özelliklerinin zaman içinde nasıl değiştiği gösterilmiştir. Bu çalışmada 2014 yılından sonra determinizm, laminarite ve entropi gibi YNA ölçütlerinin istikrarlı bir düşüş sergilediği gösterilmiştir. Bu düşüş 2014'ten sonra döviz kuru piyasasının daha öngörülemez, daha düzensiz ve daha kararsız hale geldiğini göstermektedir.	In this work USD/TRY and EUR/TRY Exchange rates are analyzed with nonlinear and chaotic time series analysis methods. In this work to detect chaos, methods such as correlation dimension, Lyapunov exponent, and surrogate data testing are utilized and obtained evidence for chaos in these exchange rates. Additionally, by utilizing recurrence quantification analysis (RQA) and cross recurrence quantification analysis (CRQA) it is demonstrated how chaotic properties of the exchange rates change through time. In this study, it has been shown that RQA measures such as determinism, laminarity, and entropy exhibited a steady decline after 2014. This decline indicates that the exchange rate market has become more unpredictable, more irregular, and more unstable after 2014.
Anahtar Kelimeler: Döviz Kurları, Kaos Teorisi, Lyapunov Katsayısı, Vekil Veri Testi, Yineleme Niceleme Analizi	**Keywords:** Exchange Rates, Chaos Theory, Lyapunov Exponent, Surrogate Data Testing, Recurrence Quantification Analysis
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Araştırma ve Yayın Etiği Beyanı	Bu makale, bilimsel araştırma ve yayın etik kurallarına uygun olarak hazırlanmıştır.
Yazarların Makaleye Olan Katkıları	Yazarın makaleye katkısı %100'dür.
Çıkar Beyanı	Yazarlar açısından ya da üçüncü taraflar açısından çalışmadan kaynaklı çıkar çatışması bulunmamaktadır.

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1. Introduction

The study of complex systems has gained importance in recent literature. There are many examples of complex systems in biology, neurology, linguistics, sociology, and economics. Some complex systems demonstrate chaotic behavior.

Economics is essentially a social science. However, in the historical development of economics, ideas and concepts are transferred to economics from fields of physics such as thermodynamics. With the applications of methods from physics in economics, a heterodox interdisciplinary research field called econophysics has emerged. Chaos theory is a field that originated from physics but spread to social science fields including economics. This study is an application of chaos theory in economics. Pioneering works of Stutzer (1980), Benhabib and Day (1981), and Day (1982) demonstrated the usefulness of chaos theory in economic analysis. After these studies, many studies appeared applying chaos theory in economics (Grandmont, 1985; Boldrin and Montrucchio, 1986; Deneckere and Pelikan, 1986; Grandmont, 1986; Grandmont and Laroque, 1986; Farmer, 1986; Chiarella, 1988; Puu, 1991; Bala et al., 1998; Mitra, 2001). Guegan (2009) surveyed the usage of chaos theory in economics and finance.

Although chaotic systems are defined with differential and difference equations precisely it is impossible to predict their long-term behavior because small differences in initial conditions amplified exponentially. Chaotic time series look random but their data generating mechanism is deterministic. Fluctuations in a time series can be explained in two ways. Fluctuations in a time series can be a result of stochastic dynamics or nonlinear chaotic dynamics. In stochastic systems, fluctuations are the results of exogenous shocks. However, in nonlinear chaotic systems fluctuations are created endogenously.

In the literature, there is an effort to detect chaos in times series from different fields. The symptoms of chaos are positive maximum Lyapunov exponent, fractal dimension, and nonlinearity. Chaos theory originated in meteorology with Lorenz (1965). There are two kinds of tools to detect and analyze chaos named metric and topological tools. Metric tools include BDS (Brock-Dechert-Scheinkman) test (Brock et al., 1996), correlation dimension, Lyapunov exponent, and surrogate data testing. Topological tools include Recurrence Plot (RP) and Recurrence Quantification Analysis (RQA).

In our paper, we have two research questions. Our first research question is whether USD/TRY and EUR/TRY exchange rates are chaotic. To answer this research question, we utilized methods such as correlation dimension, Lyapunov exponent, and surrogate data testing. We answered this research question affirmatively. Our second research question is that given USD/TRY and EUR/TRY exchange rates are chaotic, how do their chaotic properties change through time. To answer this research question, we utilized RQA and Cross Recurrence Quantification Analysis (CRQA) in our study.

The existence of chaos in economics has important consequences from both a theoretical and a practical point of view. From the theoretical point of view existence of chaos implies that it is possible to model the economic phenomena with mathematical models for example with the difference or differential equations. From a practical point of view existence of chaos implies that it is impossible to control the system in the long run. But to control the system in the short run, different intervention strategies must be adopted for chaotic systems. Chaos in economic time series also implies that neoclassic economic theory's reductionist approach is not suitable for economics. Therefore, chaos theory improved the understanding of economics and offers new approaches to the analysis of the economy.

Our paper is organized as follows: In part two we reviewed the literature. In part three we presented the description of methods we used in our study. These methods include phase space reconstruction, correlation dimension, Lyapunov exponent, surrogate data testing, RQA, and CRQA. In part four we presented applications of methods reviewed in part there. In part five we conclude our study.

2. Literature Review

In the literature, chaos has been extensively investigated in financial and economic time series. In these investigations mostly correlation dimension, BDS test, and Lyapunov exponent are used. This literature is surveyed in Faggini (2014, 2019). Literature investigating chaos in exchange rates is summarized in Table 1.

Table 1: Literature investigated chaos in exchange rates

Year	Author(s)	Tools	Results
1992	Bajo-Rubio et al.	Grassberger-Procaccia test and Lyapunov test	Evidence for chaos
1996	Sewell et al.	BDS test	Evidence for chaos
1997	Serletis and Gogas	BDS test, NEGM test and Lyapunov test	Evidence for chaos
2001	Gilmore	Close returns test	No evidence of chaos
2002	Bask	Lyapunov test	No evidence of chaos
2002	Belaire-Franch et al.	Recurrence Plot and Recurrence Quantification Analysis	Evidence for chaos
2003	Schwartz and Yousefi	BDS test, correlation dimension, and Lyapunov exponent	No evidence of chaos
2007	Torkamani et al.	Correlation dimension and Lyapunov test	Evidence for chaos
2007	Das and Das	Lyapunov exponent and surrogate data	Evidence for chaos
2009	Liu	BDS test, Lyapunov test, and surrogate data	Evidence for chaos
2010	Adrangi et al.	BDS test, correlation dimension, and entropy test	No evidence of chaos

In the literature, there are several papers that analyze how RQA measures change during the time utilizing the sliding window approach (Bastos and Caiado, 2011; Piskun and Piskun, 2011; Moloney and Raghavendra, 2012; Soloviev et al., 2020). Piskun and Piskun (2011) examined how RQA laminarity of major world stock market indexes changed over time and demonstrated that laminarity measure can be used as a tool to reveal, monitor, analyze and predict economic crises, crashes, and financial bubbles. Moloney and Raghavendra (2012) investigated phase transition in the Dow Jones Industrial Index from a bull market to a bear market using RQA. Authors demonstrated that when the scaled variance and uncertainty are rising the determinism and predictability of markets collapse. In these cases, the market loses its deterministic structure and behaves in a random manner. Soloviev et al. (2020) demonstrated that in complex economic systems, RQA measures can be employed as indicators and precursors of critical events such as crises in the economy. Authors showed that RQA measures such as LAM, DET, Vmean, Lmax, and ENTR are sensitive to economic crises in history. During the economic crises, these measures drop significantly.

Our study makes a significant contribution to the literature. There is literature investigating chaos in economic time series. However, in this literature, chaos in USD/TRY and EUR/TRY exchange rates are not investigated. Our study is the first study investigating chaos in USD/TRY and EUR/TRY exchange rates. Our second contribution to the literature is the application of the RQA and CRQA to the USD/TRY and EUR/TRY exchange rates to demonstrate how chaotic properties are changed through time. Our study is the first study applying RQA and CRQA to the USD/TRY and EUR/TRY exchange rates. Therefore, our study fills a gap in the existing literature.

3. Preliminaries

3.1. Phase Space Reconstruction

Deterministic nonlinear time series analysis methods are initialized with phase space reconstruction by an embedding process. According to Takens' (1981), embedding theorem reconstruction of phase space by time-delay embedding procedure preserves the system's original attractor from the one-dimensional output of the system. Initially, there is one-dimensional time series which is observed from the system as below:

$$\mathbf{x} = (x_1, x_2, x_3, \dots, x_n) \tag{1}$$

To reconstruct phase space of $\mathbf{x}$, two parameters named as time delay τ and embedding dimension *D* must be determined. Therefore, the first vector of reconstructed phase space $\mathbf{X}_1$ becomes:

$$\mathbf{X}_1 = (x_1, x_{1+\tau}, x_{1+2\tau}, \dots, x_{1+(D-1)\tau}) \tag{2}$$

In this way coordinates of reconstructed phase space can be expressed as a matrix below:

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \vdots \\ \mathbf{X}_{n-(D-1)\tau} \end{pmatrix} = \begin{pmatrix} x_1 & x_{1+\tau} & \cdots & x_{1+(D-1)\tau} \\ x_2 & x_{2+\tau} & \cdots & x_{2+(D-1)\tau} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n-(D-1)\tau} & x_{n-(D-2)\tau} & \cdots & x_n \end{pmatrix} \tag{3}$$

To determine time delay τ and embedding dimension *D*, false nearest neighbors and mutual information methods are proposed in the literature (Huffaker et al., 2017). However, Zbilut (2005) proposed that for economic time series, time delay τ can be taken as 1 and embedding dimension D can be taken as 10 empirically. In this work, we followed Zbilut's (2005) proposal and adopted these parameter values. This selection is in line with the works of Strozzi et al. (2007), Strozzi et al. (2008), Bastos and Caiado (2011) and Xing and Wang (2020).

3.2. Correlation Dimension

To qualify a system as chaotic, its fractal dimension must be fractional (noninteger). A fractal is identical to itself on all dimensions since it includes an infinite number of copies of itself. The number of variables required to define the system is determined by the fractal dimension.

A fractal dimension can be calculated in a variety of ways, such as the Hausdorff dimension, information dimension, correlation dimension, box-counting dimension. However, these fractal dimension calculation methods do not always give the same result. The correlation dimension, which is suggested by Grassberger and Procaccia (1983), possesses some advantages such as quick implementation and straightforwardness.

Correlation sum is a metric that quantifies the ratio of points separated by a small distance, and defined as below:

$$C(N,m,\varepsilon) = \frac{1}{N(N-1)} \sum_{m \le t \ne s \le N} \Theta\left(\varepsilon - \|X_t - X_s\|\right) \quad \varepsilon > 0 \tag{4}$$

In the statement above Θ represents Heaviside function and ‖.‖ represents norm operator.

The correlation function determines the likelihood that the distance between a pair of randomly selected points is less than ε. The correlation dimension is calculated from the correlation sum. To calculate correlation dimension, it must be specified how $C(N,m,\varepsilon)$ changes as ε changes. When ε is increased, $C(N,m,\varepsilon)$ increase because the number of nearby points increases. Grassberger and Procaccia (1983) demonstrated that for small ε, $C(N,m,\varepsilon)$ increase at rate D_c. Therefore, the following approximation can be written:

$$C(N,m,\varepsilon) \approx \varepsilon^{D_c} \tag{5}$$

So, the correlation dimension can be calculated from the following limit:

$$D_c = \lim_{\varepsilon \to 0} \frac{\log C(N,m,\varepsilon)}{\log \varepsilon} \tag{6}$$

As seen above correlation dimension depends on the choice of embedding dimension, selection of radius ε, and the norm operator. Brock (1986) and Kugiumtzis (1997) found that the Euclidian norm generates the most reliable results.

3.3. Lyapunov Exponent

Although chaotic systems have deterministic time evolution it is impossible to predict the far future of these systems. In chaotic systems, small changes in initial conditions are amplified exponentially. The Lyapunov exponent is a value that measures a system's sensitivity to initial conditions. Since sensitivity to initial conditions is a hallmark of chaos, the Lyapunov exponent can be used to detect chaos in a dynamical system. If a system has sensitive to initial conditions, trajectories that are initially close will be separated with exponential rates. However, if the system is chaotic this divergence will not explode and trajectories will remain in a bounded set. This divergence of trajectories is measured by the maximal Lyapunov exponent. According to the sign of the Lyapunov exponent, initially, infinitesimally close trajectories can converge or diverge.

Methods for estimating the Lyapunov exponent falls into two classes. These are direct methods and Jacobian methods. Both methods are based on embedding procedure. The idea under these methods is to track two nearby points and determine the divergence rate of the trajectories from these points.

The direct methods are presented by Wolf et al. (1985) and Rosenstein et al. (1993) and are based on tracking the divergence rate of nearby points. The method of Rosenstein et al. (1993) is presented below:

This method begins with time delay embedding of initial time series. After that, each point's nearest neighbor is determined. In this process nearest neighbor of reference point X_j is denoted as $X_{\hat{j}}$ and following expression can be written:

$$d_j(0) = \min_{X_{\hat{j}}} \left\| X_j - X_{\hat{j}} \right\| \tag{7}$$

In the expression above $d_j(0)$ denotes the initial distance between j^{th} point and its nearest neighbor, and ||·|| indicate the norm. By averaging the divergence rate of the nearest neighbors, the largest Lyapunov exponent can be determined. Therefore j^{th} nearest neighbor pair of points separated with a rate of maximal Lyapunov exponent as below:

$$d_j(i) \approx C_j e^{\lambda_1 (i \cdot \Delta t)} \tag{8}$$

In the expression above C_j denotes the initial separation of the nearest trajectories. If it is taken the logarithm of both sides, the following equation is obtained.

$$\ln d_j(i) \approx \ln C_j + \lambda_1 (i \cdot \Delta t) \tag{9}$$

The previous expression denotes roughly parallel lines (for $j = 1,2, \dots, M$) each having a slope approximately proportional to λ_1. Then the maximal Lyapunov exponent can be calculated by the least-squares method by an average line denoted as below:

$$y(i) = \frac{1}{\Delta t} \langle \ln d_j\,(i) \rangle \tag{10}$$

In the expression above $\langle \cdot \rangle$ indicates the average for all j.

3.4. Surrogate Data Testing

Surrogate Data Testing is a method to determine whether the data is linear or nonlinear (Schreiber and Schmitz, 2000). Both deterministic chaotic and linear stochastic data are seemed to be irregular but their data generating mechanisms are very different. Surrogate data tests involve resampling of original time series by bootstrapping methods. However, during this resampling, some parameters are fixed. In surrogate data testing null and alternative hypotheses are defined as below:

H_0 (Null hypothesis): Data is generated by a linear process.

H_1 (Alternative hypothesis): Data is generated by a nonlinear process.

The surrogate data method, which is presented by Theiler et al. (1992), involves the steps below:

1. The null hypothesis H_0 compatible with the generation process of the observed time series is determined.

2. Generate a set of resampled series consistent with H_0. These series are different implementations of the hypothetical process and are called surrogate series.

3. A discriminating statistic is computed and the distribution of this statistic is obtained.

4. Obtained value of discriminating statistic from original series compared with the distribution of discriminating statistic obtained from surrogate series by using a significance test.

For example, assume that we want to test a time series for nonlinearity. In this case, the null hypothesis is formed as “data is generated by a linear process”. According to this null hypothesis, several Gaussian linear series is generated. A suitable statistic is estimated from the original series and each surrogate and it is investigated whether this observed statistic is significantly different from surrogates.

If there is a significant difference between these statistics, the null hypothesis which assumes linearity is rejected and can be concluded that original time series data is not generated by a linear process.

In this surrogate data testing procedure several discriminating statistics such as Lyapunov exponent, entropy, and correlation dimension. In this work, we used the time reversibility statistic as a discriminating statistic. The time symmetry statistic measures the asymmetry of a time series under time reversal by calculating the following expression:

$$E[s_n \cdot s_{n+1}^2] - E[s_n^2 \cdot s_{n+1}] \tag{11}$$

Linear stochastic time series are symmetric beneath time reversal. Therefore, this statistic can be utilized to test linearity.

Surrogate data testing procedure tests whether the series is generated by following the ARMA process:

$$x_t = \sum_{i=1}^{M} a_i x_{t-i} + \sum_{i=0}^{N} b_i \varepsilon_{t-i} \tag{12}$$

The Fourier power spectrum of the surrogate series and original series are the same. This is accomplished by performing a Fourier transform on the original time series, randomizing the phase, and obtaining surrogates by inverting the transform. (Theiler et al., 1992; Schreiber and Schmitz, 2000).

3.5. Recurrence Plot (RP) and Cross-Recurrence Plot (CRP)

A recurrence plot is a visual tool that shows recurrences (repetitions) of the time series in reconstructed phase space (Eckmann et al., 1987). If the distance between two coordinates in different time indices is smaller than a threshold parameter *T* then we called this a recurrence or repetitions. We express these repetitions in a two-dimensional matrix whose rows and columns correspond to time indices. Therefore, the RP matrix can be expressed as below:

$$RP_{ij} = \Theta\left(T - \left\|X_i - X_j\right\|\right) \tag{13}$$

In the statement above Θ represents the Heaviside function which translates distances greater than T to 0 and smaller than T to 1.

In RP diagonal and vertical (horizontal) lines have special meanings. In an RP a diagonal line observed when $RP_{i+k,j+k} = 1$ for $k = 1 \dots l$. These diagonal line patterns mean that similar states lead to a similar future so it reflects predictability of the dynamics. Also, in an RP a vertical (horizontal) line is observed when $RP_{i,j+k} = 1$ ($RP_{i+k,j} = 1$) ($k = 1, \dots, v$). These patterns are observed if a state changes very slowly or does not change. In other words, the system is trapped in a state for some duration.

Cross Recurrence Plot (CRP) is similar to recurrence plot but it compares two different time series instead of one and reveals recurrences of two-time series (Marwan et al., 2002).

To plot CRP first two time series with equal length are embedded with the same parameters one time series is plotted on the x axis and the other time series is plotted on the y axis. CRP can be represented by the following expression:

$$CRP_{ij} = \Theta\left(T - \left\|X_i - Y_j\right\|\right) \tag{14}$$

As in RP, we investigate vertical, horizontal, and diagonal line patterns in CRP. Unlike RP, CRP is not symmetric around the main diagonal.

RP and CRP methods can be applied successfully to short and nonstationary data.

3.6. Recurrence Quantification Analysis (RQA)

Since RP is a visual analysis tool interpretation of an RP involves the subjectivity of the observer. Sometimes it is hard to interpret RP. Two overcome this subjectivity RQA is developed (Zbilut and Webber, 1992; Marwan and Kurths, 2002; Zbilut, 2005). In RQA simple pattern recognition algorithms are applied to RP and some measures are calculated. Some RQA measures are based on diagonal lines and some of them are based on vertical (horizontal) lines. Calculations of RQA measures are presented below:

Determinism (DET) is a measure that takes into account diagonal lines and reflects the predictability of the system. Here deterministic means similar present leads to similar future. So, the present determines the future. It is known that in RPs short and absent diagonal lines reflect stochastic dynamics and long diagonal lines reflect deterministic dynamics. Determinism measure is calculated as below:

$$\mathbf{DET} = \frac{\sum_{l=l_{min}}^{N} lP(l)}{\sum_{l=1}^{N} lP(l)} \tag{15}$$

In the expression above $P(l)$ denotes frequency distribution of diagonal lines with length l.

The length of the longest diagonal line (Lmax) RQA measure reflects the stability of the system. A high longest diagonal line length means a more stable system. The inverse of this measure reflects the maximal positive Lyapunov exponent. The length of the longest diagonal line is stated as below:

$$\mathbf{Lmax} = \max\left(\{l_i; i = 1, \dots, N_l\}\right) \tag{16}$$

In the expression above the number of diagonal lines represent with N_l.

The mean length of the diagonal lines (Lmean) is reflected by the diagonal lines' average length. This measure can be thought of as mean prediction time This measure can be expressed as below:

$$\mathbf{Lmean} = \frac{\sum_{l=l_{min}}^{N} lP(l)}{\sum_{l=l_{min}}^{N} P(l)} \tag{17}$$

Laminarity (LAM) is defined as the percentage of points in RP which constitute vertical lines. This pattern indicates that state dynamics change very slowly or do not change et al. Laminarity reflects laminar phases (intermittency) in the system and shows chaos-chaos transitions. Laminarity can be calculated as below:

$$\mathbf{LAM} = \frac{\sum_{v=v_{min}}^{N} vP(v)}{\sum_{v=1}^{N} vP(v)} \tag{18}$$

In the statement above frequency distribution of the vertical lines of lengths, v is denoted with $P(v)$.

Trapping time (Vmean) RQA measure indicates the average length of the vertical lines. This measure shows the average time that the system is trapped in a state. Trapping time is calculated as below:

$$\mathbf{Vmean} = \frac{\sum_{v=v_{min}}^{N} vP(v)}{\sum_{v=v_{min}}^{N} P(v)} \tag{19}$$

The recurrence rate (REC) measure reflects the fraction of recurrence points. It shows the probability of recurrence in the system. The recurrence rate is calculated as follows:

$$\mathbf{REC} = \frac{1}{N^2}\sum_{i,j=1}^{N} RP(i,j) \tag{20}$$

Shannon entropy (ENTR) reflects the distribution of diagonal line segments. This measure reflects the complexity and diversity of deterministic dynamics in the system. If ENTR value is high this means a high complexity and diversity and if ENTR value is low this means a low complexity and diversity. The Shannon entropy is calculated as below:

$$\mathbf{ENTR} = -\sum_{l=l_{min}}^{N} p(l)\ln p(l) \tag{21}$$

In the statement above $p(l)$ denotes the probability that a diagonal line has exactly length l.

3.7. Cross Recurrence Quantification Analysis (CRQA)

In CRQA measures such as determinism (DET), the average length of diagonal lines (L) and recurrence rate (RR) are calculated (Coco and Dale, 2014; Wallot and Leonardi, 2018; Wallot, 2019). These CRQA measures are calculated similarly to their RQA counterparts.

In CRQA if the two time series visit the same phase regions then longer diagonals are obtained. Otherwise, short diagonals are obtained. Diagonal lines show synchronization between two series. Average diagonal line length reflects the duration of synchronization between two series. If the two series much more resemble each other diagonal structures are increased.

4. Application

In this work, we analyzed USD/TRY and EUR/TRY exchange rates with nonlinear time series analysis methods. Our data spans 03-01-2005 and 27-11-2020 and consist of daily values. In the embedding procedure, we set the embedding dimension to 10 and the time delay to 1. In our analysis, we utilized the sliding window method. We set the window size to 400 days and the window step to 50. All calculations are performed using R software. Time series graphs of USD/TRY and EUR/TRY exchange rates are shown in Figures 1 and 2 respectively.

By using the method of Grassberger and Procaccia (1983) calculated correlation sum and correlation dimension values for different radiuses and embedding dimensions are given in Figures 3 and 4 for USD/TRY and EUR/TRY respectively.

By using the method proposed by Rosenstein et al. (1993) maximal Lyapunov exponents are estimated. According to this method estimated maximal Lyapunov exponents are 5.47076 and 4.099307 for USD/TRY and EUR/TRY respectively. Since maximal Lyapunov exponents are positive, the system is sensitive to initial conditions. This finding is a sign of chaos. These estimates are obtained from average slopes of lines fitted to divergence graphs shown in Figures 5 and 6.

We also carry out surrogate data testing. As seen in Figures 7 and 8 for both USD/TRY and EUR/TRY exchange rates statistics obtained from original data are significantly different from surrogate data statistics. Therefore, we reject linearity for both USD/TRY and EUR/TRY exchange rates and assume nonlinearity.

Change in determinism (DET) for USD/TRY and EUR/TRY exchange rates are shown in Figures 9 and 10. After the period between 31-8-2012 and 14-3-2014 determinism tends to fall. This means predictability is decreased after this period. For the USD/TRY exchange rate minimum determinism value is observed on dates between 4-10-2019 and 23-3-2018. And for EUR/TRY exchange rate minimum determinism is observed on dates between10-8-2018 and 21-2-2020.

Change in length of the longest diagonal line (Lmax) for USD/TRY and EUR/TRY exchange rates are shown in Figures 11 and 12. As we stated above a high longest diagonal line length means a more stable system. Figures 11 and 12 reveal that stability is decreased after the period 29-3-2013 and 10-10-2014. This inference is compatible with a change in determinism.

Changes in mean length of the diagonal lines (Lmean) for USD/TRY and EUR/TRY exchange rates are shown in Figures 13 and 14. This measure reflects the mean prediction time of the system. When we look at these figures there are peaks at the period between 12-3-2005 and 22-9-2006 in both series. There is another peak in the USD/TRY exchange rate at the period between 3-2-2012 and 16-8-3013. EUR/TRY graph shows two additional peaks at the period between 5-3-2010 and16-9-2011 and also between 22-6-2012 and 3-1-2014.

Change in laminarity for USD/TRY and EUR/TRY exchange rates are shown in Figures 15 and 16. Like determinism, laminarity shows the tendency to fall after the period between 31-8-2012 and 14-3-2014. This means the irregularity becomes intense after this date. Also, at dates between 23-3-2018 and 4-10-2019, there is a sharp decrease in laminarity which reflects the highest irregularity of changes in prices.

Changes in trapping time for USD/TRY and EUR/TRY exchange rates are shown in Figures 17 and 18. For the USD/TRY exchange rate, trapping time has a maximum value at periods between 23-5-2005 and 1-12-2006 and also between 16-8-2013 and 3-2-2012. After the second maximum value, trapping time tends to decrease. For EUR/TRY exchange rate, trapping time has a maximum value at periods between 14-3-2005 and 22-9-2006. In addition to this maximum trapping time value, there are additional two peaks at periods between 29-5-2009 and 10-12-2010 and between 25-11-2011 and 7-6-2013. After the second peak, trapping time tends to decrease. Compatible with determinism and laminarity after the period between 31-8-2012 and 14-3-2014 trapping times decrease. This reflects that the series become irregular after this date. Trapping time figures are similar to the mean length of the diagonal lines' figures.

Change in Shannon entropy for USD/TRY and EUR/TRY exchange rates are shown in Figures 19 and 20. As seen in figures Shannon entropy decreased after the period between 31-8-2012 and 14-3-2014. This means that complexity is reduced after that period. Entropy takes minimum values at the period between 23-3-2018 and 4-10-2019. This is also the minimum of laminarity.

In Figure 21 determinism of USD/TRY exchange rate and central bank reserve are compared. To make a meaningful comparison we normalize each series with z-scores. As seen from the figure there is downward co-movement after the period between 31-8-2012 and 14-3-2014 in both series. Also, a lagged relationship is seen between determinism and central bank reserve after that period.

In Figure 22 laminarity of the USD/TRY exchange rate and central bank reserve are compared. Again these two series are normalized with z-scores. As in Figure 21, there is a downward co-movement of laminarity and central bank reserve after the period between 31-8-2012 and 14-3-2014 in Figure 22.

In this work, we also carry out CRQA. We investigated the changes in CRQA measures such as recurrence rate (RR), determinism (DET), and the average length of diagonal lines (L). Change in CRQA measure recurrence rate is shown in Figure 23. A high recurrence rate means similar dynamics between series. Change in CRQA measure determinism is shown in Figure 24. High values of determinism reflect the synchronization of the two time series. Change in CRQA measure of the average length of diagonal lines is shown in Figure 25. This CRQA measure reflects the duration of the synchronization of the two time series.

5. Results and Conclusion

For both USD/TRY and EUR/TRY exchange rates, we obtained positive maximal Lyapunov exponent estimates. This finding indicates sensitivity to initial conditions and constitutes evidence for chaos. Also, our application of surrogate data tests indicates nonlinearity in both exchange rates. This is another evidence of chaos. Since the calculation of a single correlation dimension is difficult and involves subjective judgment, we did not calculate single correlation dimensions. We only supplied correlation sum and correlation dimension graphs for different embedding and radius values in Figure 3 and Figure 4.

Whether there is chaos in exchange rates has important consequences for theoreticians and policymakers. Detecting chaos in the economy adversely effects the validity of the neoclassical theory and the results of neoclassical intervention policies. Since in chaotic systems there is sensitivity to initial conditions prediction of chaotic systems, in the long run, is impossible. This fact is referred to as the butterfly effect. The main result of the chaos in exchange rates is that exchange rates can only be affected by intervention policies in the short run. In the long run, it is not possible to influence exchange rates with interventions. Chaotic properties of exchange rates negatively affect the success of neoclassical intervention policies. The existence of chaos in exchange rates necessitates different intervention strategies for policymakers to regulate and stabilize the market. Also, the existence of chaos in USD/TRY and EUR/TRY exchange rates means that neoclassic economic theory’s reductionist approach is not suitable for the analysis of these exchange rates. Therefore, our results are valuable and have implications for theoreticians and policymakers.

Our results indicate that after 2014 both USD/TRY and EUR/TRY exchange rates become more unpredictable, more irregular, more unstable, and more random. This finding is compatible with the general situation of the Turkish economy. After this date, the general situation of the Turkish economy deteriorated. This observation is compatible with the increase of Turkey's risk premium after that date. When we look at the change in laminarity for USD/TRY and EUR/TRY exchange rates (Figure 15 and 16) in the period between 23-3-2018 and 4-10-2019 a collapse is observed. This collapse period corresponds to local peaks in USD/TRY and EUR/TRY exchange rates (Figure 1 and Figure 2). In line with the literature (Piskun and Piskun, 2011; Moloney and Raghavendra, 2012; Soloviev et al., 2020), this collapse reflects a critical state and indicates a crisis.

As seen from Figures 21 and 22 reserves of the central bank of Turkey decreased steadily after 2014. This decrease can be a result of central bank intervention to stabilize the exchange rates. However, after 2014 both regularity (laminarity) and predictability (determinism) declined with central bank reserve. Therefore, it can be said that possible intervention of the central bank in the foreign exchange market could not stabilize and regulate the market.

When CRQA measures are evaluated, there is no clear significant difference between before and after 2014 as in RQA. This shows that the interaction between the two exchange rates is independent of the Turkish economy.

For future studies, chaos can be investigated in different exchange rates by using presented methods such as correlation dimension, Lyapunov exponent, surrogate data testing. Also, RQA and CRQA can be applied to different exchange rates. In this study, we utilized daily data. For future studies data with higher time frequencies can be investigated.

Figure 1: USD/TRY exchange rate

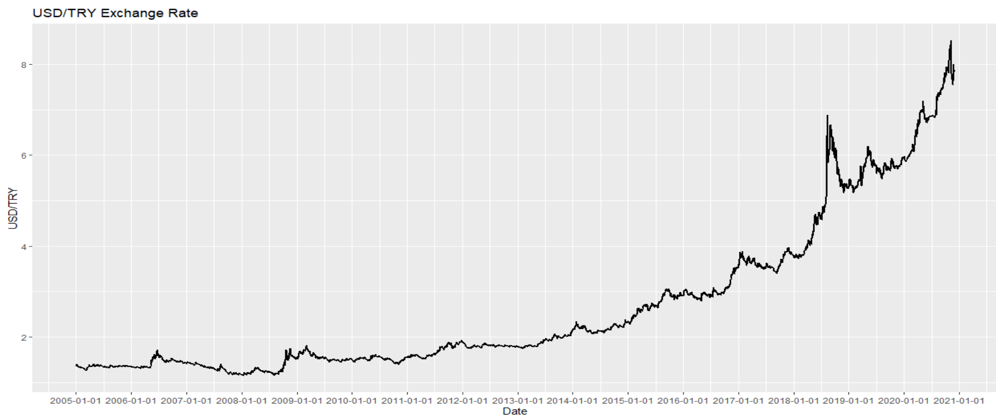


Figure 2: EUR/TRY exchange rate

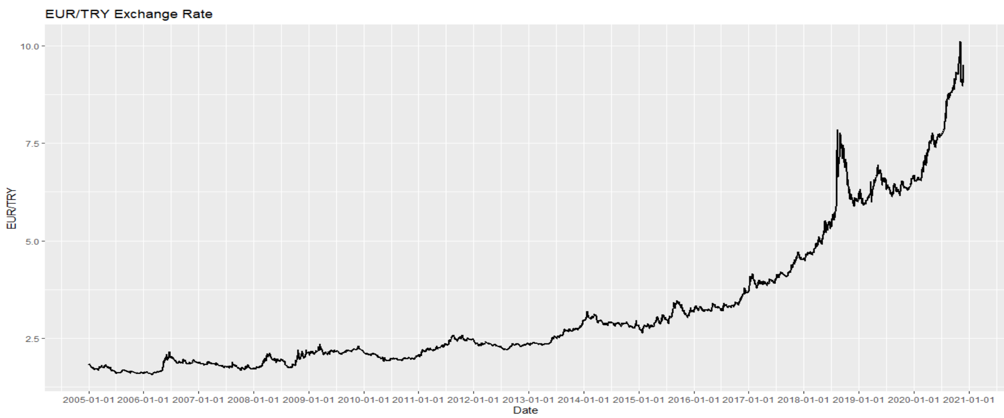


Figure 3: Correlation sum and correlation dimension for USD/TRY exchange rate

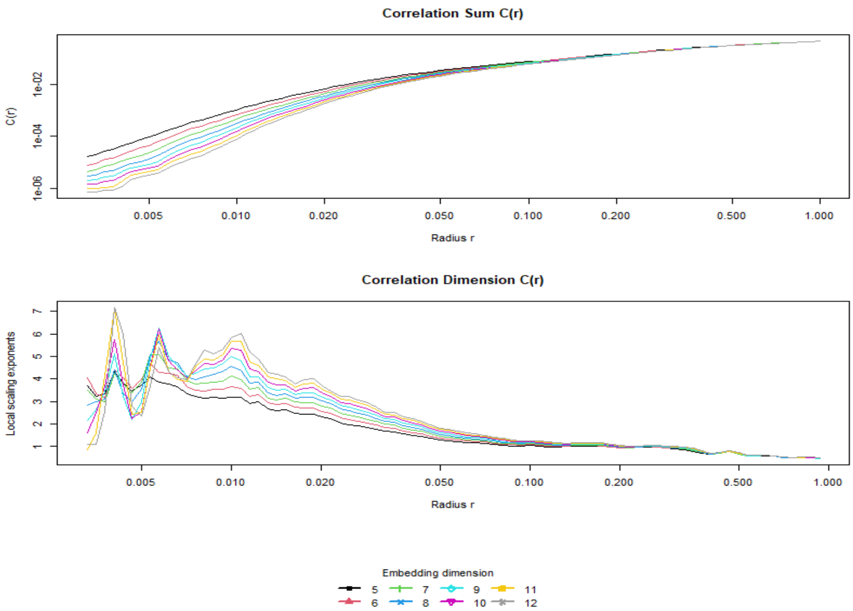


Figure 4: Correlation sum and correlation dimension for EUR/TRY exchange rate

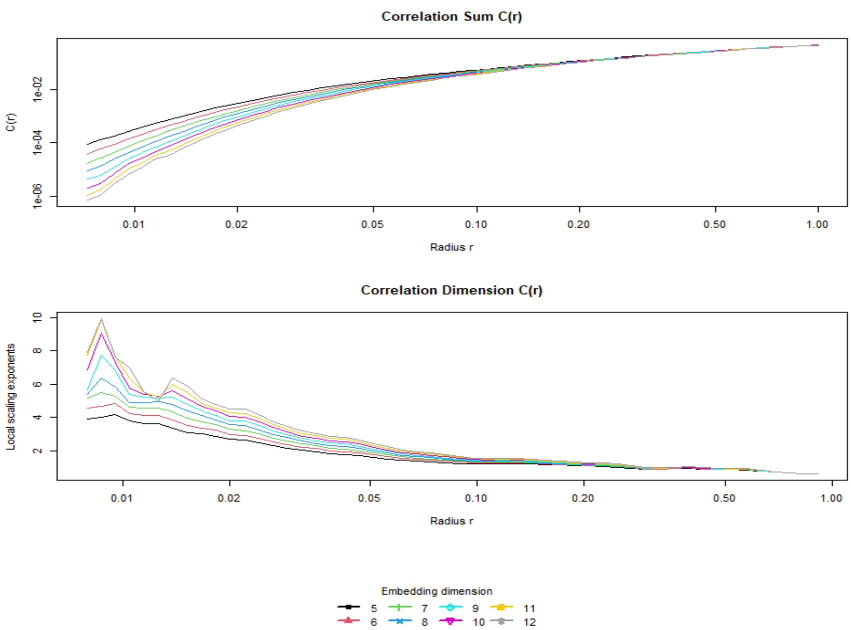


Figure 5: Maximal Lyapunov exponent estimation for USD/TRY exchange rate

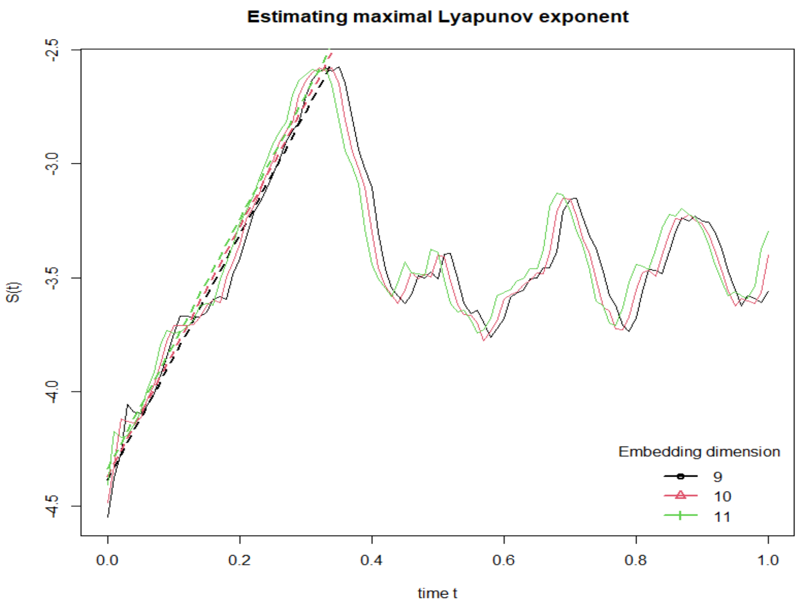


Figure 6: Maximal Lyapunov exponent estimation for EUR/TRY exchange rate

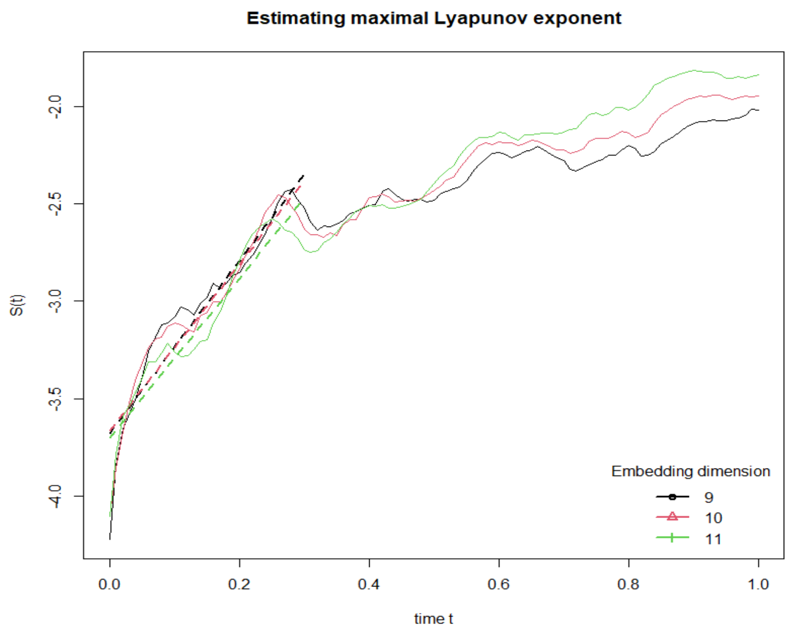


Figure 7: Surrogate data testing for USD/TRY exchange rate

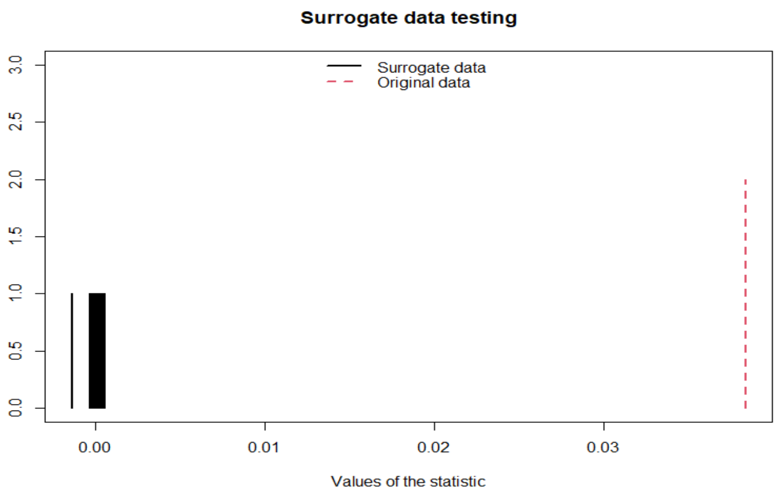


Figure 8: Surrogate data testing for EUR/TRY exchange rate

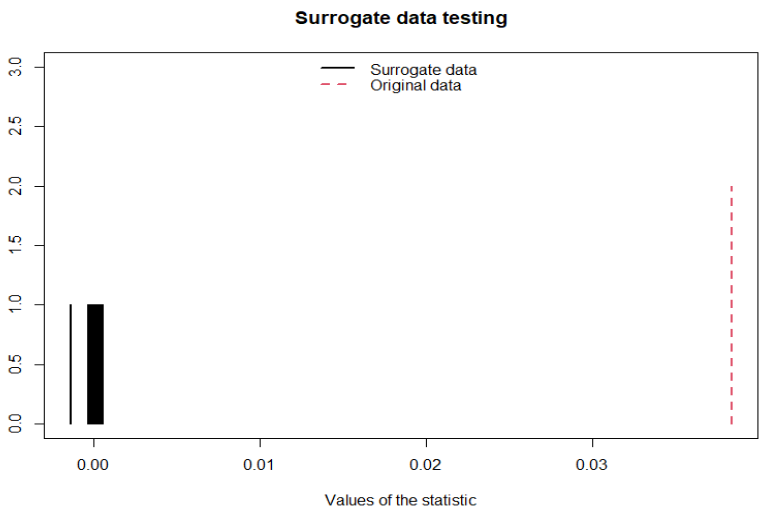


Figure 9: Change in RQA determinism for USD/TRY exchange rate

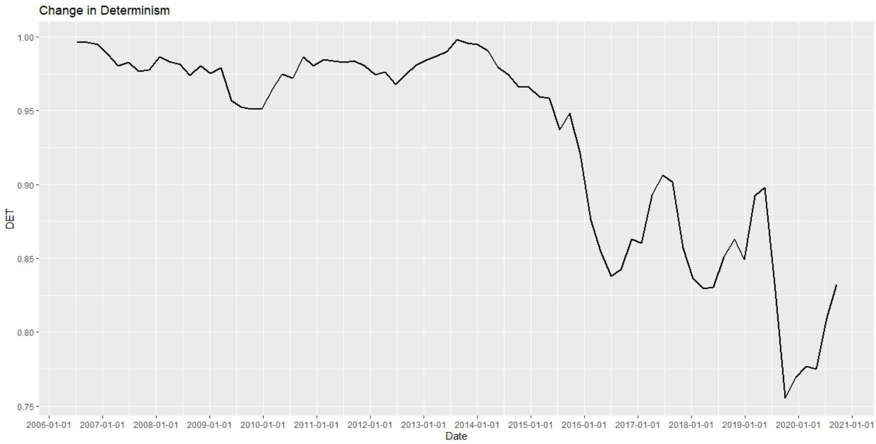


Figure 10: Change in RQA determinism for EUR/TRY exchange rate

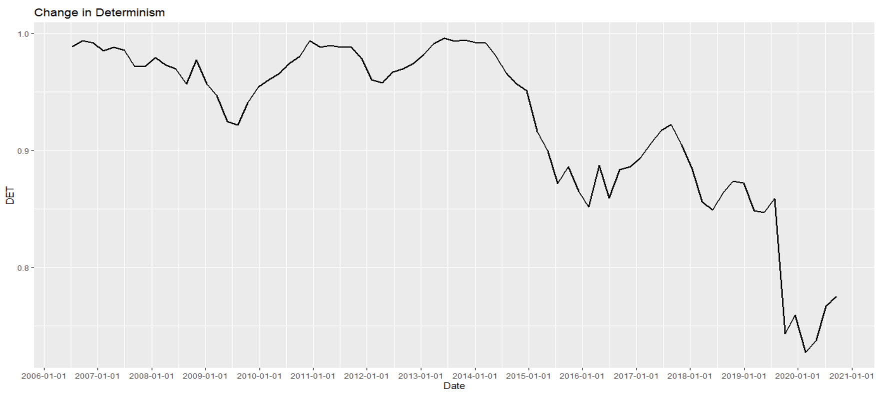


Figure 11: Change in RQA longest diagonal line length for USD/TRY exchange rate

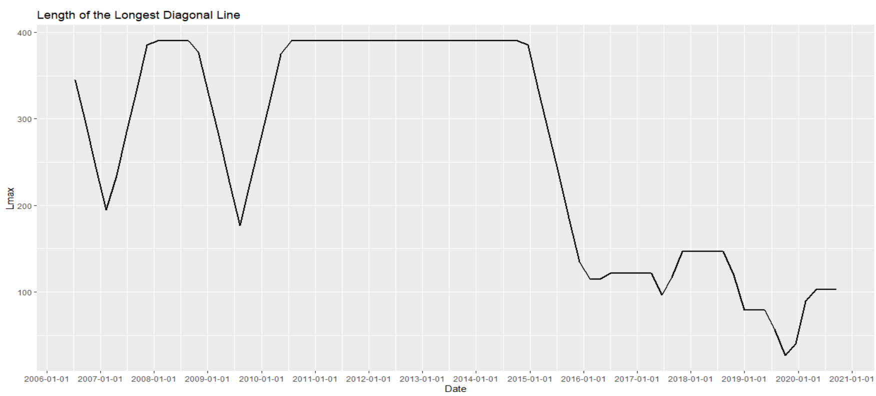


Figure 12: Change in RQA longest diagonal line length for EUR/TRY exchange rate

Figure 13: Change in RQA mean length of the diagonal lines for USD/TRY exchange rate

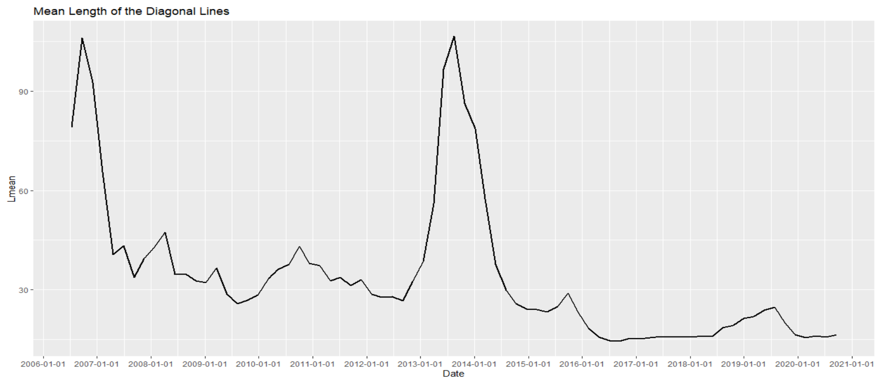


Figure 14: Change in RQA mean length of the diagonal lines for EUR/TRY exchange rate

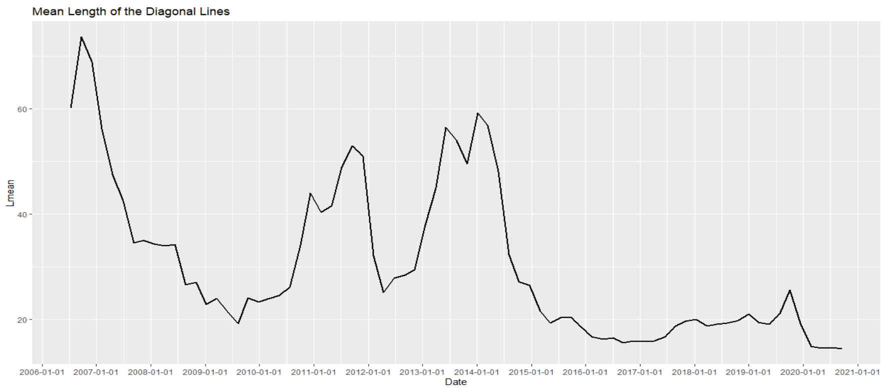


Figure 15: Change in RQA laminarity for USD/TRY exchange rate

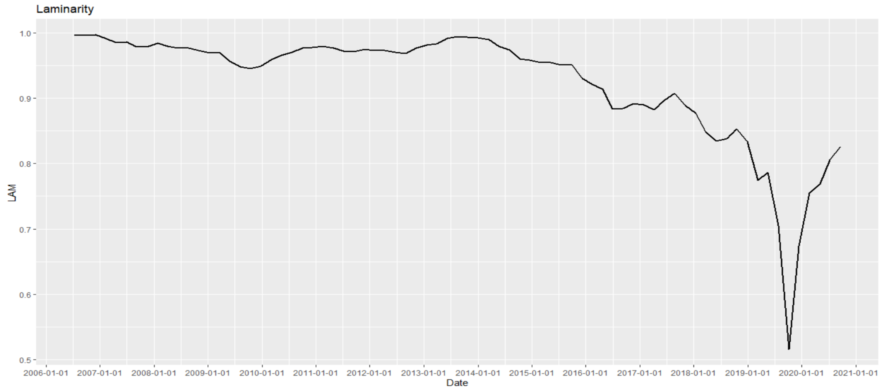


Figure 16: Change in RQA laminarity for EUR/TRY exchange rate

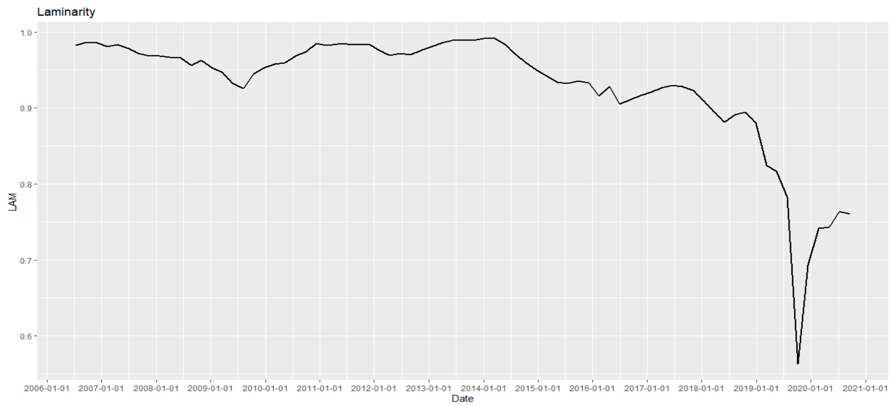


Figure 17: Change in RQA trapping time for USD/TRY exchange rate

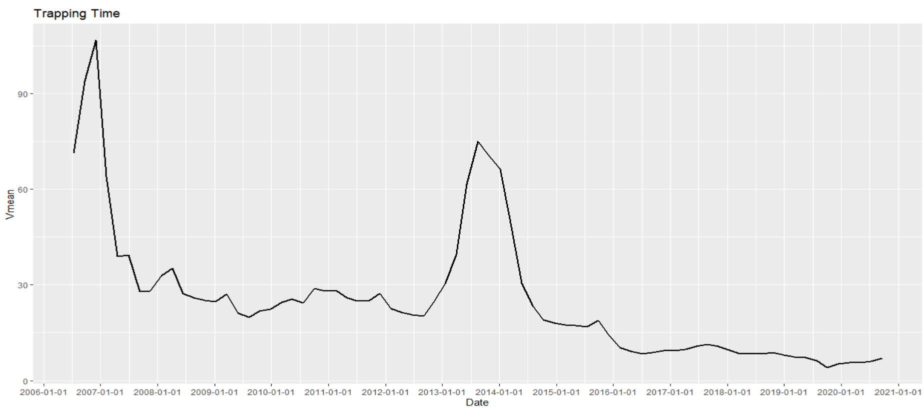


Figure 18: Change in RQA trapping time for EUR/TRY exchange rate

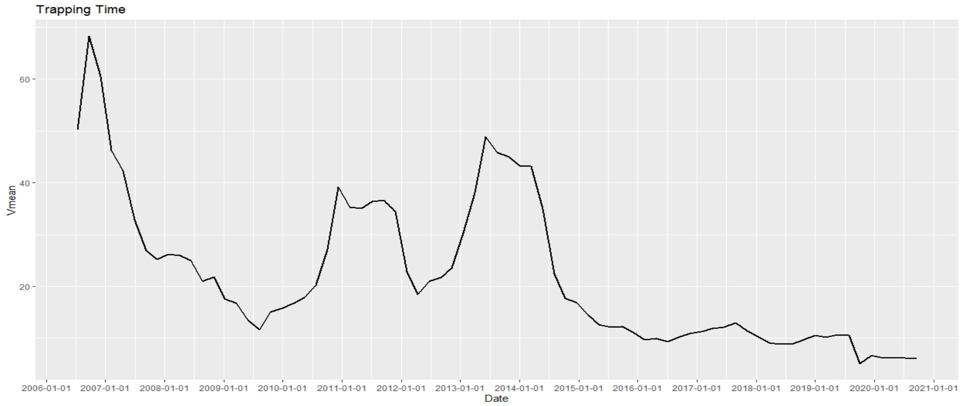


Figure 19: Change in RQA Shannon entropy for USD/TRY exchange rate

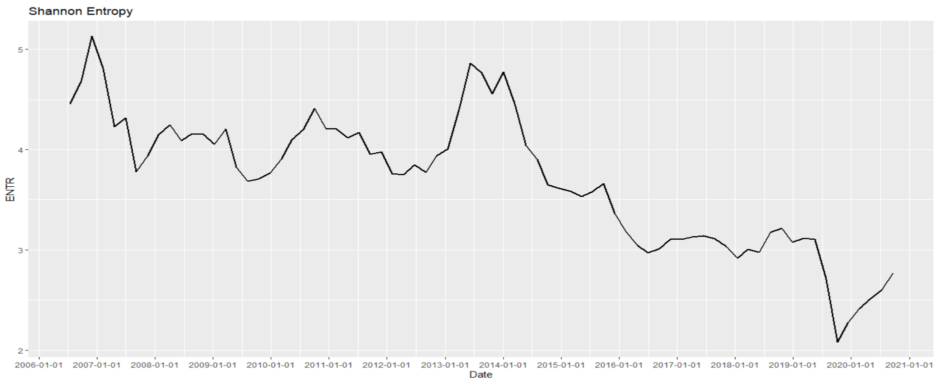


Figure 20: Change in RQA Shannon entropy for EUR/TRY exchange rate

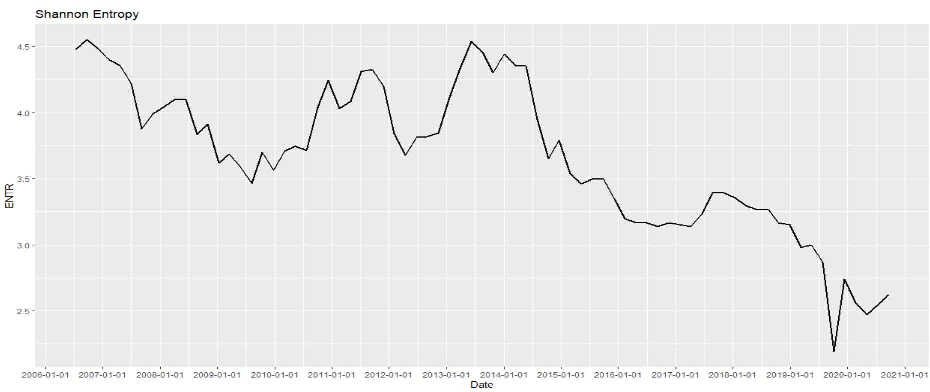


Figure 21: Change in RQA determinism and central bank reserve. Red line denotes central bank reserve and blue line denotes determinism

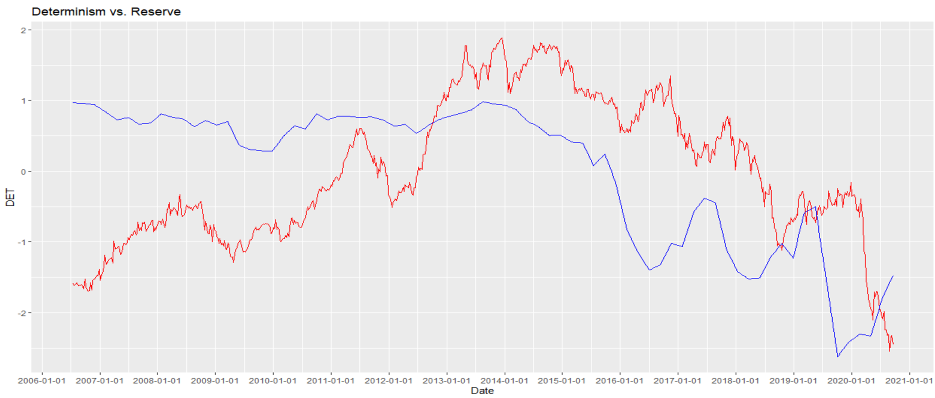


Figure 22: Change in RQA laminarity and central bank reserve. Red line denotes central bank reserve and blue line denotes laminarity

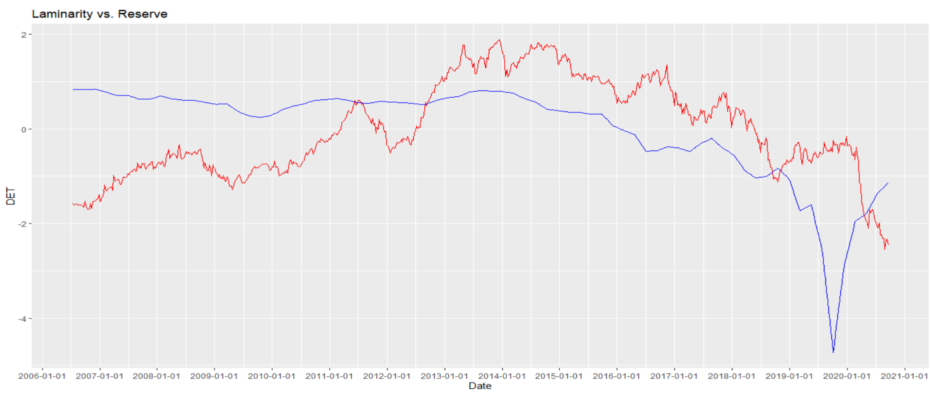


Figure 23: Change in CRQA recurrence rate

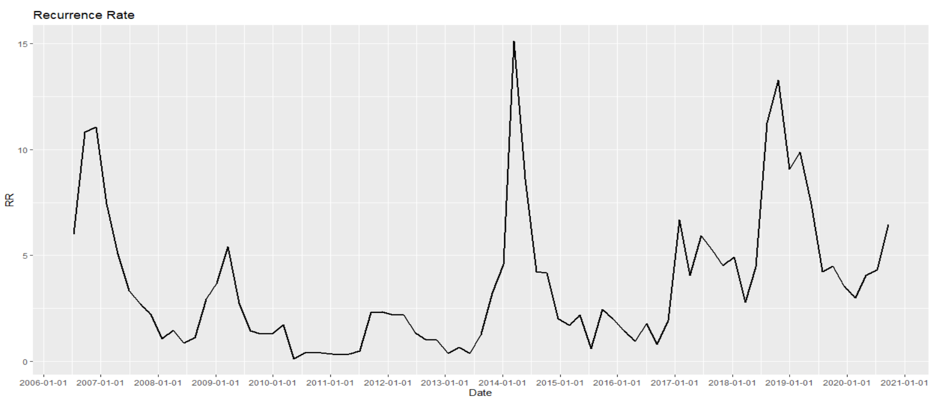


Figure 24: Change in CRQA determinism

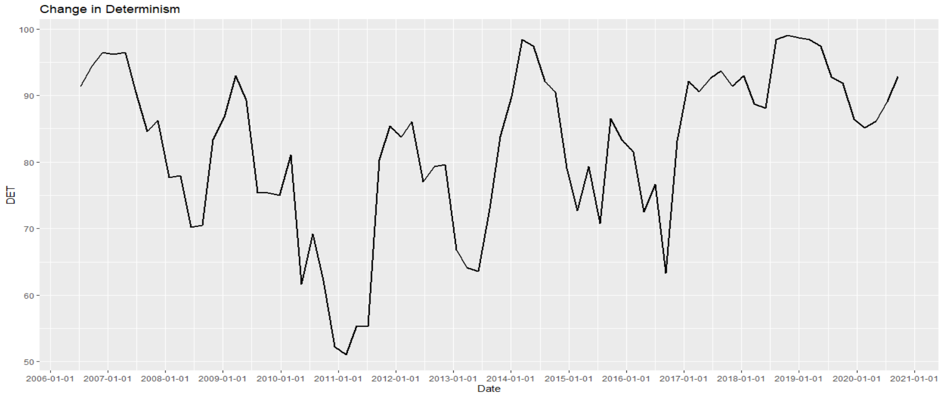
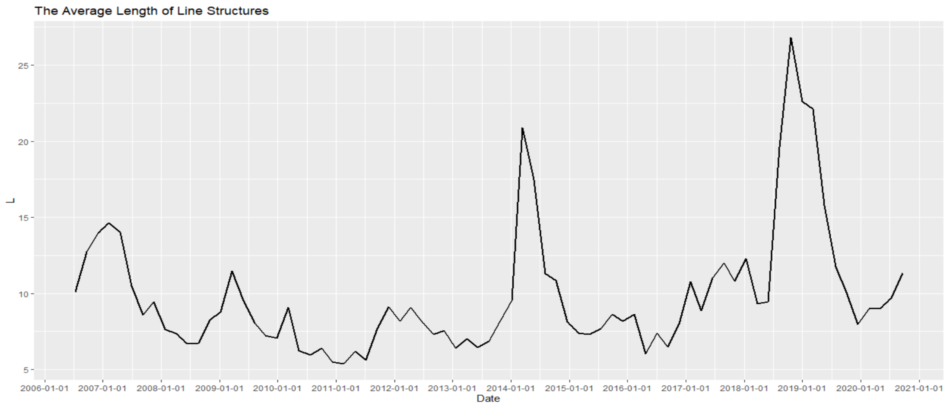


Figure 25: Change in CRQA average length of diagonal lines



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